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Artificial Snowfall from Mountain Clouds

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Abstract

A tentative theory of provoking snowfall from simple orographic clouds is composed, using simplifying assumptions, and it is shown reasonable to suppose that winter snowfall on Central Swedish mountains might be substantially increased by skillful seeding of supercooled mountain clouds.

1. Introduction

During the winter in Scandinavia extensive low clouds often occur which are only several hundred metres thick and which give no precipitation. The clouds are composed of supercooled droplets, but the temperature is not low enough for an abundant natural formation of ice crystals, which could lead to the development of snow. Nor do the clouds contain persistent vertical motions of a magnitude sufficient to sustain any considerable precipitation, except in localities where the air stream containing the clouds flows over mountains. The possibility arises, as envisaged by BERGERON (1949), of introducing ice crystals into the clouds to the windward of mountain ranges, and so provoking snowfall over the mountains.

This possibility is examined in the following paragraphs. First, the rate of growth of a crystal formed in the cloud, and the likelihood of its settling upon the mountain, are considered. Next an estimate is made of the concentration of crystals corresponding to the greatest possible snowfall rate, and the practicability of producing this concentration is discussed. Estimates are made of the frequency of occasions in Jämtland suitable for seeding operations, the amount of snow which they might provide during a whole winter, and their cost. Finally, the problem of assessing their efficiency is considered.

2. The growth of ice crystals by diffusion

The rate of growth of an ice crystal by diffusion of vapour is usually represented by the equation

$$\frac{dm}{dt} = 4\pi SC\,k\triangle p \qquad (1)$$

where m is the mass of the crystal,

S is a factor depending upon the shape of the crystal (the electrical capacity, equal to $2\,r/\pi$ for a disc and to r for a sphere, r being the radius),

C is a velocity-coefficient expressing the increase of condensation due to the (settling) motion of the crystal through the environment,

k is the coefficient of diffusion of water vapour in air,

and $\triangle p$ is the difference between the vapour densities in the environment and at the crystal surface.

If the crystal grows in a supercooled cloud the vapour density in the environment is function only of the temperature. The vapour density at the crystal surface is not that corresponding to ice saturation at the air temperature, because the crystal is warmed a little by the latent heat of condensation. However, it may readily be shown that this effect causes the vapour density excess to become the difference $\triangle p'$ between the saturated vapour densities over liquid water and ice at the air temperature, reduced by a factor $(1 + kf)$ where f is a function dependent only upon the air temperature. The coefficient of diffusion k is a function of both air pressure and temperature, but in considering crystal growth at levels near mountain tops (about 1,000 m) we may consider the pressure to be 900 mb, and then the quantity $k\triangle p$ in equation (1) becomes F, equal to $k\triangle p'/(1 + kf)$ and a function of temperature only, as shown in Table 1:

is less than about 50 μ, but that this fraction increases to 10 or more for radii up to about 200 μ, the thickness of large crystals rarely exceeding 30—40 μ, and often lying in the range of 5—20 μ for large dendritic crystals of diameter up to about 1 mm or even more.

Appropriate values for the fall-speed and the coefficient C are equally uncertain. For spheres Frössling found

$$C = 1 + 0.23\,(Re)^{\frac{1}{2}}, 2 < (Re) < 800$$

where (Re) is the Reynolds number. KINZER and GUNN (1951) confirmed this relation for freely falling and evaporating droplets over a range of (Re) from about 10^2 to 10^3, but found that the coefficient 0.23 in the relation was not constant at lower values, increasing to a maximum of about 0.45 for (Re) of 4, and then decreasing to practically zero for (Re) less than about 0.5. For discs and with $(Re) > 500$, POWELL (1940) found a velocity coefficient about equal to that for spheres; the coefficient

Table 1.

Temperature, °C:	—2	—4	—6	—8	—10	—12	—14	—16	—18
$10^8 \cdot F = 10^8\, k \triangle p'/(1 + kf)$ g/cm sec:	1	2	2.5	3.1	3.4	3.8	3.7	3.6	3.4

The other terms in the equation are less satisfactorily handled, because of the variety of forms of ice crystals. When they are grown in supercooled clouds they have the form of hexagonal plates at temperatures between 0° C and — 5° C; at rather lower temperatures prisms and hexagonal columns occur, and between — 9° C and — 25° C plane dendritic and stellar crystals, and hexagonal plates are found. Crystals which grow while settling through air of a variety of temperatures have complex forms and often show intricate patterns of hollows and protruding arms, so that their bulk density may be as little as 0.5.

In the following paragraphs we shall consider the hexagonal plate to be the form typical of the range of temperatures concerned (about — 6 to — 15° C), and use a value of S equal to $2r/\pi$, regarding the crystal as equivalent to a circular disc of the same mean radius r. It is then necessary to consider the relation between m and r, or between r and the thickness a of the crystal. From the few available observations (SCHAEFER, 1947; REYNOLDS, 1952; MASON, 1953; and OKITA and KIMURA, 1954) it appears that $(2r/a)$ is about 3 when r

for discs in the range of (Re) covered during crystal growth (about 1 to 50) is unknown, but can be assumed to be given by the Frössling equation.

For (Re) of about 1 or less the fall-speed of columnar or plate-like crystals is approximately the fraction 0.7 of the fall-speed of spheres of equal volume and density (MCKNOWN and MALAIKA, 1950). SCHAEFER (1947) observed the relation between the mass and fall-speed of crystals for values of (Re) exceeding about 5, from which it appears that plates of thickness 40 μ, and diameter 400 μ and 1 mm have fall-speeds of about 40 and 65 cm/sec respectively. These values are considerably higher than the value of about 30 cm/sec found by NAKAYA and TERADA (1935) for large plane dendritic crystals of diameter 1.5 to 6 mm, whose thickness was only about 10 μ. However, observations by the same authors on hexagonal plates and capped hexagonal columns gave fall-speeds varying between 32 and 95 cm/sec, with an average value of about 55 cm/sec.

In view of all the above uncertainties we introduce some simplifications. The first stages in the crystal growth we regard as best represented by the experimental observations of REYNOLDS (1952), who produced crystals of diameter up to about 200 μ by dry-ice seeding of a cloud supercooled to $-18°$ C in a large cold box. It is not possible to produce larger crystals by this technique, since at this size the fall-speed reaches about 10 cm/sec and the crystals are soon precipitated from the cloud. The crystals settle upon the floor of the box in a variety of sizes; under these conditions hexagonal plates and plane dendritic crystals of mean diameter 150 μ and thickness 20—35 μ were grown in a period of about 90 sec. The largest of the observed crystals reached this size in about 70 sec. These growth-periods are considerably faster than would be deduced from equ. (1), assuming $C = 1$. MASON (1953), also found that small hexagonal plates (of diameter up to 50 μ) grown at $-2°$ C in a supercooled cloud had a growth rate about 50 % higher than that predicted by the equation. This discrepancy may be due to the presence of the cloud droplets, which, as pointed out by MARSHALL and LANGLEBEN (1954), act as nearby sources of vapour not envisaged in the simple diffusion theory.

We ignore the temperature variation of the quantity F, and adopt a mean value of 3.2×10^{-8} g/cm sec as appropriate to the range of temperatures (-6 to about $-15°$ C) in which we are interested. We consider the crystal growth in three stages. In the first, the radius reaches 75 μ in the period of 90 sec observed by Reynolds. At the end of this stage we assume the thickness to be 30 μ, and calculate a fall-speed of about 12 cm/sec ($Re \approx 1.1$). The assumed value for the growth period, t_1, is supported by SCHAEFER (1953), who observes that crystals grown in the supercooled cloud of a cold-box at -10 to $-15°$ C attain diameters of 100—200 μ and fall-speeds of 10—50 cm/sec in less than a minute.

Evidently during the subsequent growth the thickness hardly changes, since plate crystals of thickness exceeding about 40 μ are apparently not found. We therefore assume that in the second stage of growth, during a period t_2, the thickness of the crystal increases uniformly to 40 μ as the radius increases to 200 μ. At this size the fall-speed calculated from the drag coefficient of a disc, and assuming a mean density of 0.7, is 33 cm/sec. However, beyond this size there is a characteristic slipping and fluttering motion during the fall of the crystal which probably results in a higher mean fall-speed, and in view of this and the value of about 30 cm/sec found by Nakaya and Terada for crystals of thickness only about 10 μ, we assume the mean fall-speed to remain constant, during the third growth stage, at the value of 40 cm/sec indicated by Schaefer's data.

The growth period during the second stage (radius 75 μ to 200 μ) is then obtained from the simplified equation

$$\frac{dm}{dt} = 8\, r\, C \times 3.2 \times 10^{-8}$$

together with some assumption concerning the velocity coefficient C and the relation between the mass and radius of the crystal.

Since during this period (Re) varies from 1.1 to about 10 we may assume for C a constant value of approximately 1.6. Assuming, also, that the thickness a is given by

$$a = 24 \times 10^{-4} + r/12.5$$

we have

$$m = 0.7\,\pi r^2\,(24 \times 10^{-4} + r/12.5) \text{ and}$$
$$dm = 0.7\,\pi\,(48 \times 10^{-4} r + 3\, r^2/12.5)\, dr$$

Equ. (1) then becomes

$$dt = \frac{0.7\,\pi\,(48 \times 10^{-4} + 3\, r/12.5)\, dr}{8 \times 1.6 \times 3.3 \times 10^{-8}}$$

which is readily integrated to give t_2, the period during which the radius increases from 75 to 200 μ, as

$$t_2 = 540 \text{ sec}$$

During the third stage the thickness remains 40 μ, so that

$$m = 0.7\,\pi r^2 \times 4 \times 10^{-3} \text{ and}$$
$$dm = 1.76 \times 10^{-2}\, r\, dr$$

Also, since the fall-speed remains 40 cm/sec

$$(Re) \approx 480\, r$$

and $C \approx 1 + 0.23\,(Re)^{1/2} = 1 + 5\, r^{1/2}$

Equation (1) then becomes

$$dt = \frac{1.76 \times 10^{-2}}{8 \times 3.2 \times 10^{-8}} \cdot \frac{dr}{1 + 5\, r^{1/2}}$$
$$= 6.9 \times 10^{+4}\,(1 + 5\, r^{1/2})^{-1}\, dr$$

This equation may be integrated to give the period t_3 required for the growth to any particular size from an initial radius of 200 μ; the period for the entire growth is $(t_1 + t_2 + t_3)$, and is shown in Table 2, together with the distance settled relative to the air during the growth, calculated from the growth periods and the mean fall-speeds.

Table 2.

Mass, μg	Radius of droplet of same mass, μ	Radius of crystal, μ	Time, sec	Fall path, m
		200	630	180
14	150	400	1,400	480
32	200	600	2,000	740
56	240	800	2,600	970
88	275	1,000	3,200	1,200
127	310	1,200	3,700	1,400
172	345	1,400	4,200	1,600
225	380	1,600	4,600	1,770

We may remark that the values in this table are not seriously affected by changes in the shape assumed for the growing crystal. Thus, for example, if after the first growth-stage the crystal thickness is allowed to remain at 30 μ then the values in the last two columns are decreased by about 20 % and 30 % respectively (since both C and the fall-speed are reduced, while the crystals grow more rapidly). However, over much the greater part of the growth periods the calculations represent what can be regarded as a minimum likely growth rate. Particularly when the crystals are large, the growth rate may be greater by a substantial fraction, partly on account of underestimated velocity coefficients, and partly because of the presence of neighbouring vapour sources in the form of cloud droplets.

3. Results of experimental seeding of clouds

The results of several series of experiments in the seeding of layer clouds with dry ice in general support the values of Table 2, although they suggest that the development of snow and the fall of snow crystals may be more rapid than indicated by the table.

Langmuir (1948 b) describes the seeding (with dry ice) of a stratocumulus with base at 1,500 m (— 3° C) and top at 2,500 m (−9° C). Snow began to fall from the cloud 1,600 sec after the seeding; after 2,500 sec the investigating aircraft flew through the precipitation and found it to consist of "rather large snowflakes". After 2,800 sec these snowflakes had fallen to a level nearly 1 km below the cloud base; these values would be in accord with those in Table 2 if the crystals had originated at the cloud base level, but since in fact they formed and grew at considerably higher levels both the growth rate and fall-speed must have been greater than assumed in the calculations. The discrepancy may be partly due to the aggregation of individual crystals into clusters (snowflakes) of higher mean fall-speed. It will later be shown that crystal concentrations of the order 10^4 to $10^5/m^3$ are to be expected; such concentrations imply a mean crystal separation of only 2 to 5 cm, so that small variations in fall-speed and sideways motions are likely to result in crystal collisions and aggregation. A rough calculation shows that chance aggregates of 2 or 3 crystals are likely to collide with single crystals after a fall of about 50 m. Langleben's (1954) observations indicate that an aggregate of eight plate-crystals of radius 200 μ has a fall-speed of 60—100 cm/sec, and one of eight crystals of individual radius 600 μ a fall-speed of 70—110 cm/sec. The increased values of fall-speed above the constant value of 40 cm/sec assumed for individual crystals strongly favours continued aggregation, and causes the snow to descend about twice as rapidly as shown in Table 2, after about the first 500 seconds of growth. The adjusted values would be in better agreement with the results of the seeding experiment described by Langmuir.

Smith (1949) has discussed the result of seeding with dry ice three examples of altocumulus (layer) clouds and two examples of small cumulus. Snow trails of individual small crystals fell from two of the altocumulus clouds, of thickness 300 and 450 m, beginning 550 and 700 sec after the seeding. The third cloud had a thickness of 600 m (temperature at base — 2°, at top — 7° C); snow streaks were first seen 1,000 sec after seeding and after 1,300 sec the whole cloud had become transformed to ice, with snow consisting of flakes up to 1 cm in diameter falling from its base. The two cumulus clouds were about 1,100 m thick (temperature at base — 3°, at top — 7°C); a small snow streak became visible below the first cloud 600 sec after it was seeded, and by 750 sec the snow had become conspicuous down to 150 m below the cloud base, containing densely packed snowflakes of 2—5 mm diameter. The snow reached the ground, 550 m below the cloud base, 1,100 sec after the seeding. The seeding of a neighbouring cloud had a similar result; 1,000 sec afterwards snow streaks extended to 300 m below the cloud base, and on flying through the snow after 1,750 sec the aircraft found it to consist of densely packed flakes 2 to 5 mm in diameter. In the first three experiments the rate of fall of the first snow was estimated at $1^1/_2$, $2^1/_2$ and 4 m/sec, while in the last two it was 4 and 3 m/sec.

SHELLARD and GRANT (1951) describe the seeding of a stratocumulus 150 m thick, temperature — 9° C, which lay above another layer of supercooled cloud 1,800 m thick. Snow fell from the seeded area and was observed by radar to descend about 2,300 m within 1,900 sec of the seeding, and was later seen to reach the ground in "not very large flakes". From the shape of the snow trails as recorded on the radar and the known horizontal wind shear the fall-speed of the snow was calculated to be 1.3 m/sec in one trail and rather less in two others.

In the course of a series of similar experiments, AUFM KAMPE, WEICKMANN and KELLY (1953) observed on two occasions the size of snow crystals which fell from seeded stratocumulus clouds. On the first occasion the snowfall from the seeded area left a hole in the cloud which was about 2 km wide 1,800 seconds after the seeding. Later, snow crystals collected from snow trails at the edges of the expanding hole were found to be hexagonal plates and stars of about 1 mm diameter. These had fallen from cloud originally 470 m thick with a temperature of about — 15° C. On the second occasion a cloud about 900 m thick (temperature at base about — 5° C, at top — 8° C) was observed to be "snowing out violently" less than 1,500 sec after seeding, and when the snow area was investigated it was again found to contain crystals of about 1 mm diameter. On another occasion snow reached the ground 1,200 sec after the seeding of a cloud with base about 500 m (— 6° C) and top about 1,400 m (— 9° C) above the ground. The average fall-speed of the first snow crystals must therefore have exceeded 40 cm/sec.

HALL, HENDERSON and CUNDIFF (1953) provide photographs showing the development of snow from a seeded orographic cloud about 1,400 m thick (temperature at base 0° C, at top — 9° C). These show that the snow fell about 500 m below the cloud base within about 600 sec of the seeding.

All these results provide a strong support for the theoretical calculations. They show that the first snow settles more than 500 metres in periods of from 1,000 to 1,900 sec after the seeding, in accordance with the second row of Table 2, but suggest that when the crystals grow in high concentration over depths of several hundred metres they become aggregated into flakes which have fall-speeds about twice as great as the 40 cm/sec assumed in deriving the subsequent entries in the table.

4. Conditions for collisions between growing crystals and cloud droplets

Since we are concerned with the growth of crystals inside supercooled clouds the possibility must also be considered of the crystals colliding with droplets, and so growing by accretion as well as by condensation. The important factors here are the concentration of droplets lying in the path of a falling crystal, and its ability to collect them. Some of the droplets are wafted aside in the air flow around the crystals, and so only a fraction E of the projected area of the crystal is effectively swept of droplets. LANGMUIR (1948 a) has shown that E, at low (Re), is a function of a parameter

$$K = 0.22\, r_C^2\, v/r\eta$$

where r_C is the radius of the droplets, η is the coefficient of viscosity of air, and r is the radius of a *spherical* collector falling through the droplet cloud at a relative speed v. It can be anticipated that a disc has a catch at least as great as that of a sphere of equal cross section, so that r and v can be identified with the radius and fall-speed of the hexagonal-plate crystal. At values of (Re) of the order 1, Langmuir has found that E is given by the relation

$$E = \{1 + 0.75 \log 2K/(K - 1.214)\}^{-2}$$

For values of K less than 1.214, the efficiency of catch is zero and the crystal cannot collide with any droplets. The calculated efficiency of catch is sensitive to the radius assumed for the cloud droplets, and some estimate must be made of the likely value.

Simple mountain clouds are formed by upward motions imparted to the air when the wind blows over the slopes, but often the orographic disturbance in the air flow is superimposed upon an already-existing system of shallow cumulus or stratocumulus clouds. Although away from the mountain such clouds may be well broken, near and over the mountain they become fused into a practically continuous layer with a considerably increased thickness. Inside such clouds, containing irregular or convective stirring motions, the concentration of liquid water increases with height above the base-level at a rate of about 1 g/m^3 km. Typical droplet concentrations in clouds formed in unpolluted air vary between about 100 and $500/cm^3$, and under these conditions the bulk of the liquid water in the clouds is contained in droplets of radius greater than about 5 μ near the base-level, 6—8 μ a few hundred metres higher, and 10 μ about 1 km above the base.

If now we calculate the values of K and E for crystals of radius 75, 200, 400 and 800 μ falling through clouds consisting of droplets of radius 6, 8 and 10 μ, we obtain the values shown in Table 3.

Table 3. Efficiency of catch E of crystals of radius r falling through clouds of droplets of radius r_C.

r_C, microns:	6		8		10	
r, microns:	K	E	K	E	K	E
75, 200....	0.96	0	1.7	0.15	2.7	0.3
400........	0.48	0	0.85	0	1.3	0.15
800........	0.24	0	0.42	0	0.65	0

We see from this table that only droplets of radius greater than 8 μ can be collected by the falling crystals. Droplets of this size in significant concentrations are likely to occur only in the upper parts of clouds at least several hundred metres thick, so that in general the accretion process is unlikely to contribute to the crystal growth. It is significant that in none of the experiments mentioned above are graupel structures, the typical accretion products, described; the snow is always found to consist of individual crystals or snow-flake aggregates. Inside clouds containing many droplets of radius greater than 8 μ, that is, inside the larger cumuliform clouds, it can readily be shown that accretion becomes effective when the crystals have radii approaching 200 μ, and then produces a substantial increase in their fall speed, so that the accretion soon makes the dominant contribution to the growth of the crystals. These then develop into roughly spherical aggregates of graupel or hail. Inside shallow layer clouds this development is unlikely to occur, and the typical precipitation is composed of individual or loosely-clustered ice crystals. The presence of the cloud droplets is important in seeding operations, however, in providing a source of crystals and in increasing their rate of growth by diffusion. They also represent a reservoir which tends to maintain saturation with respect to liquid water even in the absence of an updraught.

5. Conditions favourable for producing snowfall from mountain clouds

The rate of growth of snow crystals determines the scale of the conditions which are suitable for the artificial production of snow over mountains. The important circumstances are illustrated in Fig. 1, which represents the flow of air at a mean speed V over mountains. If clouds occur to windward of the mountains with internal temperatures lower than about $-7°$ C, conditions are favourable for the production of ice crystals by the introduction of silver iodide smoke. If they are formed at a height h above the ground it can be assumed that they must settle an equal distance relative to the air in order to reach the surface of the mountain. In the first example in the diagram they are carried by the wind beyond the mountain tops before settling out of the cloud, so that their growth is interrupted and they are subject to evaporation. In the second example the distance D between the region where the air begins to sink in the lee of the mountain and the region in which the crystals form in the mountain cloud, is so large that the crystals settle out upon the summit or windward slopes of the mountain. The condition that this should occur is seen from Table 2 to be that the

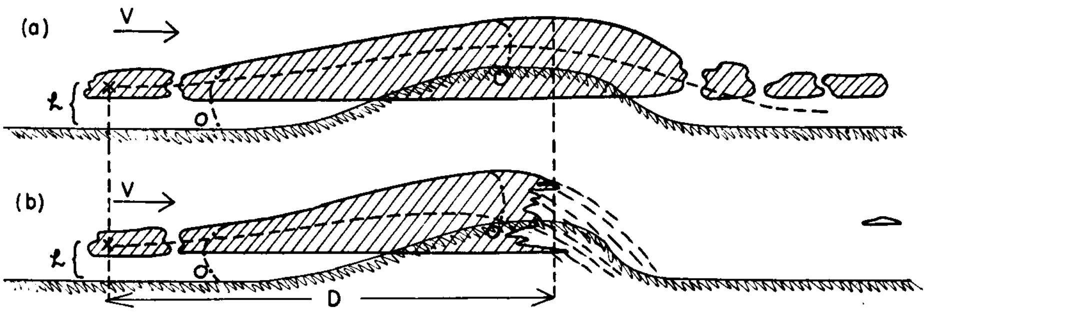

Fig. 1. Formation of snow in mountain clouds. An ice crystal introduced at a height h (several hundred m above ground) into clouds windward of the mountain has a trajectory shown by the pecked lines (partly lying within the region oo of orographic updraught). The crystal is initially a distance D from the region where air begins to descend in the lee of the mountain. In (a), (D/V), where V is the mean wind speed, is considerably less than 1,500 sec; the crystal attains a radius of less than about 200 μ, settles relative to the air less than 200 m, and evaporates in the lee of the mountain. In (b), (D/V) is rather more than 1,500 sec; the crystal attains a diameter of about 1 mm, settles relative to the air several hundred m, and thus is precipitated upon the mountain. If an appropriate concentration of crystals is introduced, each follows a similar trajectory, and the vapour condensed in the cloud is consumed in producing the snowfall, so that the mountain cloud disappears where the air begins to descend behind the mountain.

period D/V should be about 1,500 sec, assuming the height h to be several hundred metres. In winds of 5 to 15 m/sec this implies that the crystals should be introduced into the clouds at distances of some 8 to 23 km windward of the region where the air begins to descend behind the mountain. If the vapour condensed in the orographic updraught over the mountain is to be efficiently precipitated in the form of snow, the crystals should settle through at least the upper part of the region of updraught, practically completing their growth before arriving at the region of descending air.

The function of the updraught is to maintain the state of saturation with respect to liquid water, in which the snow crystals grow at the maximum possible rate. If all the water provided by the updraught, and that provided by the evaporation of the original cloud, is efficiently deposited as snow upon the mountain, then the cloud disappears in the region where descent of air begins, and the air remains clear in the lee of the mountain.

6. The concentration of snow crystals

If the crystals settling through the region of orographic updraught are to grow at the maximum rate, they must not be so numerous that the updraught is unable to maintain a state of water saturation. In air ascending under these conditions about 0.5 g/m^3 of water vapour is provided for condensation in a rise of 500 m. The rate of provision of vapour is therefore about 10^{-5} U g/m^3 sec, in an updraught of speed U cm/sec. A typical mean value of U in the region of orographic updraught is about $^1/_2$ m/sec. If the rate of vapour supply in such an updraught is equated to the rate of removal by the growth of N crystals/m^3, of radius about 500 μ, in air saturated with respect to liquid water, we obtain

$$\begin{aligned} N dm/dt &= N \cdot 8\, r\, CF \\ &= N \times 0.4 \times 2.1 \times 3.0 \times 10^{-8} \\ &= N \times 2.5 \times 10^{-8} \\ \text{but } N dm/dt &= 50 \times 10^{-5} \\ \therefore N &\approx 2 \times 10^{4} \end{aligned}$$

If the concentration of crystals introduced into the cloud should be less than this optimum value, then not all of the vapour provided by the orographic updraught is condensed upon the snow crystals; rather, the density of the droplet cloud increases towards the mountain tops, beyond which some evaporation occurs in the region where the air descends. The seeding operation does not then have the maximum possible efficiency.

On the other hand, the efficiency is not sensitive to concentrations of crystals somewhat in excess of the calculated value. If, for example, it is greater by a factor of 3, the crystal growth proceeds at the maximum rate until the crystal radii approach about 200 μ. At this stage the crystals remove vapour from an environment saturated with respect to liquid water more rapidly than vapour available for condensation is supplied by the updraught. For some while the evaporation of cloud droplets allows the rapid growth to continue, but when this evaporation is complete the vapour pressure and the growth rate fall to reach new steady values, in which the rate of increase of the crystal radii is about halved. This is not serious, however, for the crystals have already attained their maximum fall-speeds of about 40 cm/sec: the vapour provided by the orographic updraught is still efficiently deposited upon the snow crystals, and these reach the ground in about the same place, although individually they are smaller than in the instance where they have the concentration calculated above. Indeed, since under the reduced excess of vapour pressure the tabular form of growth may revert to a columnar mode, with a tendency for the fall-speed to increase, or because the increased concentration of crystals favours the formation of snowflake aggregates, the crystals may reach the ground more rapidly. It is hardly possible to say at what crystal concentration the stage of 'over-seeding' is reached, in which the crystals individually remain too small to reach the mountain surface before being carried well into its lee and subjected to evaporation. Since, however, even crystals of rather less than 200 μ radius have considerable fall-speeds (20—40 cm/sec), a crystal concentration excess by a factor of as much as 10 may not seriously reduce the efficiency of the seeding operation.

7. Performance of freezing-nucleus generators

The practicability of the seeding operation hinges on the necessity of providing crystal concentrations within the cloud of the order 10^4 to $10^5/m^3$. Very high concentrations of crystals can be introduced at all levels in a supercooled cloud by seeding from an aircraft with pellets of dry ice. Since, however, the seeding operations are imagined to occur over periods of several hours at a time, both by day and night in conditions when the mountain tops are obscured by clouds, it is clear that the method of seeding from the ground with smokes of freezing nuclei must be used. There is then some doubt whether the smoke enters the clouds in useful concentrations. Within the clouds themselves, if they consist of assembled small cumulus or of stratocumulus with practically clear skies above, there will usually be a moderately vigorous convective stirring. On at least some occasions such stirring extends to levels below the cloud base, but in their absence the turbulent stirring in moderately strong winds flowing over mountainous country must be relied upon to diffuse the smoke upwards. There are no adequate theories on which to base calculations of the rate of diffusion in a variety of likely conditions, and we can only refer to some empirical investigations carried out mainly in weather during which convection extended upwards from the ground.

In the work of BRAHAM, SEELY and CROZIER (1952), the diffusion of an aerosol of fluorescent pigment, dispersed in particles of about 1 μ radius from a ground generator at the rate of about 3×10^{10}/sec, was examined. The particle concentrations observed by an aircraft downwind of the generator were reduced to those expected with a wind speed of 11 m/sec by multiplication with the fraction $11/V$, where V was the actual wind speed. This reciprocal law was established by SUTTON (1947) for the diffusion of clouds near the ground over distances of a few hundred metres from a generator. After this correction the data showed that at heights of several hundred metres above the ground and at distances of 10—40 km from the generator the particles occurred in average concentrations of 100 to $200/m^3$. At distances beyond about 15 km the diffusion seemed to be relatively slow, suggesting that the eddies responsible for the diffusion are of scales much smaller than this (the convective motions may be expected to have dimensions of the order of 1 km or less). All the observations showed the width of the region over which concentrations of the above order were found was about half the distance from the generator. The results, which have been extended and improved by CROZIER and SEELY (1955), indicate that in a well-stirred atmosphere particles reach levels up to about

1 km above the ground at distances of about 20 km from the generator, in concentrations of the order of 10^{-8} *S* in winds of 5—20 m/sec, where *S* is the source strength in particles/sec.

In a similar investigation SMITH and HEFFERNAN (1954) used generators of silver iodide smoke, in which a solution of silver iodide in acetone was burnt in either a hydrogen or a kerosene flame. Both burners consumed silver iodide at the rate of about 200 g/hr; the hydrogen burner produced freezing nuclei active at — 7°, — 11° and — 20° to — 30° C at the rates of 5×10^{10}/sec, 5×10^{12}/sec and 10^{14}/sec respectively. The kerosene burner produced larger particles of silver iodide, and its rate of freezing nucleus generation was only about a tenth of that of the hydrogen burner. In an aircraft flown downwind of the generators samples of air were examined for freezing nuclei by chilling them in a cold box to produce supercooled clouds, and observing the numbers of ice crystals which formed. Measurements made upwind of the generator were used as a control.

The observations showed that a high proportion of the particles emitted from the hydrogen burner soon lost their efficiency as freezing nuclei: after 25 minutes exposure to the air the flux of effective nuclei was only one thousandth of the production of the burner. On the other hand the inactivation of nuclei from the kerosene burner was hardly detectable.

The phenomenon of inactivation of silver iodide nuclei has been studied by several investigators and attributed to photolysis at the surface of the particles. It has been related to exposure to sunshine and air of high humidity, but more recently BOLTON and QUERSHI (1954) appear to have found high air temperature to be the most important factor. Under the conditions of low temperature and general cloudiness in which freezing nuclei would be generated near mountain clouds in high latitudes the loss by inactivation is likely to be small. If so, it is reasonable to expect concentrations downwind of a generator using the more productive hydrogen flame to be greater by a factor of ten than those measured by Smith and Heffernan when the kerosene burner was used. Their observations show that in winds of about 10 m/sec the average concentrations of freezing nuclei active at about — 20° C, at heights of 500—800 m above ground and at distances of 10 to 15 km from the kerosene burner, were about 5×10^4/m³. The source strength was about 3×10^{12}/sec, so that the dilution was in close agreement with that observed by Braham, Seely and Crozier.

On the basis of these results an efficient hydrogen burner whose nuclei do not suffer loss by inactivation produces corresponding concentrations of 10^3/m³ active at — 7° C and 10^5/m³ active at — 11° C. Thus providing that temperatures as low as — 10° C occur in the mountain clouds it is possible for one generator to provide the desired concentrations of crystals when it is placed some 10—40 km from the clouds in which the crystals are to be formed. If the temperature inside the clouds is anywhere considerably lower than — 11° C there may be some risk of providing too many crystals. In the present state of knowledge, however, there is no reliable basis for any discussion on the important subject of how to ensure the efficient production of artificial snowfall by control of generator output and positioning according to the particular meteorological situation. It can only be said that when clouds with internal temperatures below about — 10° C occur over the windward slopes of large mountains, then it is reasonable to expect that one generator at a distance of perhaps 40 km from the upper slopes can provide sufficient freezing nuclei to support an efficient artificial snowfall over a front of 10 to 20 km. Much experimental work would be needed to improve our understanding of how best to conduct such a seeding operation in a variety of meteorological situations.

The rate of diffusion of nucleus-smokes, and the magnitude and position of the mountain updraught, may be greatly influenced by hardly apparent details in the structure of the air stream (see, e.g. CORBY, 1954). The latter are also dependent upon the mountain shape: the most pronounced effects are caused by long ranges extending across the wind; isolated mountains, even though higher, are likely to produce weaker and less extensive orographic updraughts, and may therefore offer fewer opportunities of producing artificial snowfall. Consequently only a careful survey of conditions actually within a region where seeding operations are proposed can provide satisfactory data for an estimate of their feasibility and effect.

8. Quantitative estimate of artificial snowfall from mountain clouds

The maximum quantity of snow which can be obtained by seeding a mountain cloud is approximately the amount of water vapour condensed in the orographic updraught. This can be estimated by assuming that air near the ground is displaced upwards in the updraught by an amount about equal to the height of the mountain tops above the lowlands. In a typical example this displacement might be 500 m. If the general cloud base is 500 m above the low ground the surface air is brought just to saturation at its new level (assuming the lower layers are well stirred), preserving the cloud base at the same general level over the mountains. In air originally at the cloud base level the displacement condenses about 0.5 g/m^3 of water, and the average condensation in a column 500 m high is therefore 0.25 g/m^3. If this condensation in the lowest 500 m is removed by the fall of snow through the layer, and if the wind speed is V m/sec, then the snow behind each metre of the mountain flank facing the wind falls at the rate of 125 V g/sec. If the snow is spread over a path of 10 km in the wind direction the precipitation rate is $4.5 \times 10^2\ V$ mm/hr, or about $^1/_2$ mm/hr in a wind of 10 m/sec. This corresponds to an increase in snow depth at the rate of about 5 mm/hr. If the cloud base were only 300 m above the lowground, this maximum rate of snowfall is nearly doubled, and if in addition the thickness of the swept layer is 700 m, then it is trebled.

In the lee of the mountain the air is drier than it would be if it had not provided the artificial snowfall over the mountain. If widespread precipitation is reaching the ground in the lee of the mountain, it must suffer an increased loss by evaporation, equivalent to the amount of artificial snowfall. The effect of the seeding operation is then to transfer to the mountain a proportion of the precipitation which would otherwise fall, perhaps over a relatively large area, on lower ground in the lee of the mountain. Although the total precipitation over the entire region may not be increased significantly, it may be desirable to have more of it fall at high elevations, where for example as snow it can be stored until the end of the winter. On the other hand, if the artificial snowfall is produced when there is no general precipitation over the region in the lee of the mountain, then it can be claimed as a net increase for the entire region. It is doubtful if the mean annual precipitation of the whole earth can be increased by seeding, so that the increase over the mountains can then perhaps better be regarded as balanced by a very small decrease, quite undetectable, over a large part of the earth. This larger area may of course include oceans, in which any precipitation is wasted, and regions with a superabundance of rainfall.

9. Application to mountains of central Sweden

The annual yield of artificial precipitation from mountain clouds depends upon the frequency of occasions suitable for seeding operations. Observations of the kind, height, thickness, extent and temperature of clouds over mountains are necessary if an estimate of this frequency is to be made, but are not available in the usual climatological records. It is also desirable to have information on the disturbance in the wind flow produced by the mountains and the likely rate of diffusion of smoke from a nucleus generator, and to know whether or not natural precipitation occurs at an appreciable rate generally or over the mountain. In making plans for seeding operations in a particular area it is therefore necessary to make special meteorological observations and investigations for at least one or two seasons before an estimate of their possible result can be made.

As an illustration, however, we can use observations from the mountain weather-station of Blåhammaren, which is at a height of about 1,000 m above sea level in Jämtland, central Sweden, among mountains whose tops are mainly between about 1,000 and 1,500 m. The general level of the low ground is about 400 m. Particularly in maritime air masses, these mountains are often covered by extensive low clouds with a base about 800 m above sea level, and in the winter these clouds are supercooled. In such clouds the Blåhammaren station reports fog and rime. The rime frequently occurs also during snowstorms, suggesting that the natural snowfall does not efficiently remove all the vapour condensed in the orographic updraughts. We assume, however, that conditions suitable for seeding the

Table 4. Hours of fog (without snow or with snowfall less than 0.3 mm/hr) at Blåhammaren, during months Nov.-May, 1951—53 and 1952—53.

Mean wind speed $\overline{V}$: Beaufort force	m/s	Temperature, °C: −3	−4	−5	−6	−7	−8	−9	−10	−11	−12	−13	less than −13	Total hours H, with temp. −5° C or less	$H\overline{V}$
a) 1951—52															
0	0.1			3							1			4	0
1	0.9							6						6	5
2	2.5			9	8	18	3		5			9	5	57	142
3	4.5	34	17	17	5	40		33	27					122	549
4	6.7	65	14	25	15	20	25	43	17			22		167	1,119
5	9.3	20	64	9	10	11	18	35	27	15		3		128	1,190
6	12.3	17		14	27	7	19	19	16					102	1,255
7	15.5	4	3	25	8	28		4	5					70	1,085
8	19.0			23			5						3	31	589
9	22.5									10				10	225
10	26.5						3							3	80
														$\Sigma H\overline{V}$:	6,239
b) 1952—53															
0	0.1								2					2	0
1	0.9								6					6	5
2	2.5	10		5	13	12	30		30					90	225
3	4.5	6		41	3	26	38	75	56	15			7	261	1,175
4	6.7	18	12	31	44	27	26	63	22				3	216	1,448
5	9.3	9	106	39	35	34	51	6	14	26				205	1,907
6	12.3	22		3	9	26		16	6	1	9			70	860
7	15.5		20	3		7	12	29						51	790
8	19.0	11	3			4	6				12		3	25	475
9	22.5					2								2	45
10	26.5						4						10	14	371
														$\Sigma H\overline{V}$:	7,301

mountain clouds occur when Blåhammaren has fog with a temperature of — 5° C or below, but no snow or only snowfall at an average rate estimated to be less than 0.3 mm/hr. On some of these occasions the clouds may not have extended sufficiently high (a few hundred m above Blåhammaren) to provide the temperature of — 10° C necessary for the artificial production of sufficient concentrations of ice crystals. On the other hand there are likely to have been other occasions not included but suitable for seeding operations, when the cloud base was above the station, or when the station temperature was higher than — 5° C but the cloud tops were a kilometre or more above the station and therefore at a temperature below — 10° C. Table 4 shows the distribution of hours of fog according to temperature and wind speed during the winters 1951—52 and 1952—53.

If now it is assumed that the vertical displacement over a large mountain in this region amounts to 600 m and that the cloud base is on the average 400 m above the low ground, the rate of precipitation over a 10 km stretch of the mountain tops is found in the way described in section 8 to be $10^{-1}\ V$ mm/hr. Thus the maximum possible contribution of a seeding operation to the winter snowfall is one tenth of the sum of the entries in the final columns of each part of the table. This amounts to more than 600 mm in each winter, representing a snow-depth of a few metres. The average precipitation over the Jämtland mountains during the winter is according to climatic charts between about 350 and 450 mm (during each of these winters, however, the precipitation at Blåhammaren was about 600 mm).

Some support for the estimate of the artificial snowfall can be obtained from a picture (fig.

9) in a paper by MELIN (1943). This shows a heap of rime which has fallen during a winter to the feet of a precipitation gauge at the top a 1,200 m mountain south of Jämtland. The heap appears to have a radius of about 2 m, and an average height of about 50 cm. It was collected by a gauge presenting an area of about $^1/_3$ m^2 to the wind. If it is assumed that the density of the heap and the efficiency of catch of the gauge are 0.5 g/cm^3 and 0.5 respectively, a simple calculation shows that if during riming conditions the corresponding amount of supercooled water in a column 600 m high had been removed over a path extending for 10 km along the wind, the average precipitation over the area would have been 300 mm.

It must be emphasized, of course, that this is only a tentative estimate of the maximum possible amount of the artificial snowfall, and that it could occur only over a limited mountain area, and does not represent a general increase of the same magnitude over the whole of a river catchment. The maximum amount of artificial precipitation can better be appraised by comparing its total volume with that of the spring flood in the rivers. The water-equivalent corresponding to the maximum artificial snowfall of 600 mm over a mountain chain extending 50 km across the prevailing winds is about 250×10^6 m^3. A typical river in this area with a catchment containing such a mountain chain, has a spring flood discharge of about $1,000 \times 10^6$ m^3 over a period of three months.

In the winter of 1953—54 the precipitation in Jämtland was much below normal. Weather observations are available not from Blåhammaren, but from Skurdalshöjden, which is on a hill about 20 km farther north, at a height of just under 800 m. An analysis of the fog observations, and also of records of rime deposited upon test bodies, provides an estimate of about 80 mm as the maximum artificial snowfall for this winter, or about 30×10^6 m^3 water over a large mountain group. The data are not satisfactory, however, because this station is only at about the average height of the cloud base in the winter, and so may not always experience fog when clouds suitable for seeding rest on the mountain tops. Equally the rime deposits are likely to be much smaller than those which would occur nearer the mountain tops, where the clouds contain more condensed water. The maximum artificial snowfall may be about proportional to the natural precipitation, but it is hardly possible to draw even tentative conclusions until more detailed observations are made.

10. The cost of seeding operations

In seeding operations over a mountain chain of this size a number of freezing nucleus generators would be placed at carefully chosen sites around the mountains, and it might be desirable to have some mobile generators. The cost of constructing generators is small compared with the cost of operating them, which amounts to about £ 1.3/hr. At any one time about five generators would be used, and a season's work on this scale is therefore likely to be composed of about 3,000 generator-hours with a total consumption of about 300 kg of silver and a cost of about £ 4,000. To this would be added the salaries of at least a pair of enthusiastic and skilled operators, and also minor expenditure, including the cost of transport for mobile generators and the operators, of forecast services, and of equipment and surveys needed for observing and analysing the results of the work.

Laboratory studies of the physical nature of particles in silver iodide smokes and of the process of inactivation may well lead to improvements in the design of generators and in the nucleating efficiency of their smokes, with substantial reduction of operating costs.

11. The problem of evaluating the results of seeding operations

The estimated maximum attainable increases in the mountain snow-cover and spring flood discharge of the rivers in the region considered amount respectively to about 100 % and 25 % of the normal values. Such increases would be easily noticeable. However, it must be emphasized that these estimates are tentative, and it is unlikely that present seeding techniques have a very high efficiency. More probably any artificial increases, at least at first, would be considerably smaller, and, indeed, smaller than the natural seasonal variations. There is about an even chance, for example, that the discharge in the upper reaches of the Jämtland rivers during a three-month period including the spring flood departs from the normal by more than 15 %. Moreover, for the purpose of improving seeding techniques it is very desirable to be able to evaluate even smaller effects due to changes of method, and to be able to study the results of seeding in a variety of meteorological conditions. However, because conditions suitable for seeding are those in which natural snowfall often occurs, and because the effects sought are only as large or even smaller than common fluctuations due to natural and imperfectly understood causes, the difficulty of detecting them is a major problem.

The effects of seeding can be established only by statistical methods using some kind of control for comparison with natural, undisturbed conditions. Comparison is usually made with natural behaviour in the operation area, as shown by meteorological or other records, or with the natural behaviour simultaneously observed in neighbouring, unseeded areas. The methods may be combined by using the records to compare conditions in the several areas during both periods when seeding has and has not been performed. However, any method employing historical data, especially in mountainous areas, suffers from their extreme paucity and possibly also from the existence of climatic trends. The routine observations which are made even today are usually quite inadequate to determine the amount and distribution of precipitation over mountains. This is due to technical as much as to financial difficulties. For example snow-gauges have a number of imperfections and if exposed on mountains are difficult to reach and tend in winter weather. Surveys of the depth and water-equivalent of the snow-pack are also difficult and even dangerous, and are usually regarded as providing only an index of the general snow-cover, so that many hydrologists believe that the spring river-discharge can give a better measure of the winter snowfall. On the other hand if the results of seeding operations are sought only on a seasonal basis, then each individual experiment, of the series needed for statistical investigation, requires a year to perform.

There is thus urgent need for the development of evaluation techniques, and especially for the invention of techniques for detecting the effects of seeding individual cloud systems. These will spring from new ideas which are likely of arise only from improved understanding of precipitation processes, gained from detailed observation and intensive study of cloud and precipitation systems. Only by this means can the science of cloud physics achieve its purposes of improving the prediction of the weather, and of increasing man's ability to influence, and finally to control the weather.

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